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THE EXPERIMENTAL DETERMINATION OF THE THICKNESS OF WEAK SHOCK FRONTS IN LIQUIDS

By

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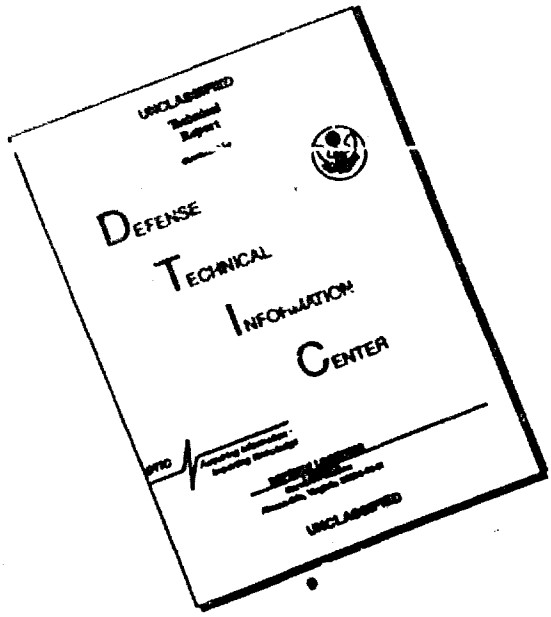

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ABSTRACT

The thickness of weak shock fronts, across which the pressure difference was in the vicinity of ten atmospheres, was measured by the reflectivity method in acetone, ethyl alcohol and ethyl ether. They are all of the order of 10^{-4} cm. thick. Lower limits to the thickness were established in water, carbon tetrachloride, benzene and ethylene glycol. A theoretical expression for the thickness is derived. Comparison between theory and experiment suggests that the effective viscosity in the shock front is somewhat greater than the ordinary shear viscosity.

THE EXPERIMENTAL DETERMINATION OF THE THICKNESS OF WEAK SHOCK FRONTS IN LIQUIDS*

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It has proved possible to investigate the transition region in shock fronts in gases by making use of the fact that the thickness of the transition region is of the same order of magnitude as the wave-length of visible light under suitable conditions of shock strength and initial gas pressure; because of this circumstance the optical reflectivity is sensitive to the thickness and density profile of the shock front.(1)

1. D. F. Hornig, Phys. Rev. 72, 179 (1947)

From their reflectivity the thickness of shock fronts has been measured in argon, nitrogen, oxygen and hydrogen and rotational heat capacity lags have been studied in the latter three molecules.(2,3,4)

2. G. R. Cowan and D. F. Hornig, J. Chem. Phys. 18, 1008 (1950)

3. E. F. Greene, G. R. Cowan and D. F. Hornig, J. Chem. Phys. 19, 427 (1951)

4. E. F. Greene and D. F. Hornig, J. Chem. Phys. 21, 617 (1953)

Very little has been published relating to the thickness of shock fronts in liquids. Cole[5]

5. R. H. Cole, "Underwater Explosions" (Princeton University Press) Princeton, N. J. 1948, p.46.

has published a summary of the work in underwater explosions, and in it he states: "A number of writers have made such estimates (of the thickness of intense shock fronts, based on hydrodynamic theory) .... the common result is .... that they predict thicknesses of the order of 10^{-5} or 10^{-6} cm ...". No detailed theoretical work has been done to date on this subject, and there has been no experimental evidence as to the thickness of any shock in a liquid. In the present paper, the thicknesses of weak shocks in liquids will be calculated from the hydrodynamic equations and an equation of state. An apparatus to measure these thicknesses will be described, and experimental results in a variety of liquids will be presented.

Shock Front Thickness in Liquids.

The three hydrodynamic equations representing the conservation of mass, momentum and energy across a plane shock front progressing in the positive X direction in a homogeneous compressible fluid may be written respectively

$$\rho u = a \tag{1}$$

$$\frac{4}{3}\mu \frac{du}{dx} = au + p - b \tag{2}$$

$$K \frac{dT}{dx} = uE + ub - \frac{1}{2}au^2 - c \tag{3}$$

where ρ is the density at any point in the fluid, u the particle velocity with respect to the shock front, μ the shear viscosity, p the pressure, E the internal energy per unit volume, K the coefficient of thermal conductivity and T the absolute temperature,

a, b and c are constants defined by the initial and final conditions. A convenient equation of state is obtained by expanding the pressure as a function of the density and the entropy:

$$\delta p = \left(\frac{\partial p}{\partial \rho}\right)_S \delta\rho + \left(\frac{\partial p}{\partial S}\right)_\rho \delta S + \frac{1}{2}\left[\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S \delta\rho^2 + 2\left(\frac{\partial^2 p}{\partial \rho \partial S}\right)_{S,\rho} \delta\rho\,\delta S + \left(\frac{\partial^2 p}{\partial S^2}\right) \delta S^2\right] \tag{4}$$

Weak shock fronts in liquids, across which the pressure difference Δp is of the order of ten atmospheres are to a good approximation isentropic. Therefore, the terms in δS in Eq. (4) may be dropped and only those in $\delta\rho$ and $\delta\rho^2$ used. This approximation is equivalent to replacing Eq. (3) by

$$dS/dx = 0 \tag{5}$$

The thickness of the shock front is defined by the expression

$$L = \frac{\rho_2 - \rho_1}{(d\rho/dx)_{max.}} \tag{6}$$

where the subscripts 1 and 2 refer to the limiting regions ahead of and behind the shock front. Eqs. (1), (2) and (4) may be solved for $d\rho/dx$ to yield, in the case of weak shocks,

$$-\frac{4}{3}\mu\frac{du}{dx} = \frac{c_0^2\rho_1}{a}\left[1 + \frac{\rho_1}{2c_0^2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S - \Delta p\left\{\frac{2}{c_0^2\rho_1} + \frac{\rho_1}{2c_0^4}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S\right\}\right]\delta\rho^2 - \frac{\rho_1\Delta p}{a}\left[1 + \frac{\rho_1}{2c_0^2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S\right]\delta\rho \tag{7}$$

In deriving this equation use has been made of the fact that $au_1 + p_1 - b = 0$ in the limiting region before the front, and that u_1, the velocity of the shock front, differs from the sound velocity C_0 by less than one part in one hundred. Use was also made of the relations

$$\left(\frac{\partial p}{\partial \rho}\right)_S = \frac{C_p}{C_v}\left(\frac{\partial p}{\partial \rho}\right)_T = c_0^2 \tag{8}$$

and

$$u_1^2 = \frac{\rho_2}{\rho_1}\,\frac{\Delta p}{\Delta \rho} \tag{9}$$

The last two terms in $\delta\rho^2$ in Eq. (7) are actually third order as a consequence of Eq. (9). For example, for a 10 atmosphere shock in ethyl alcohol, they amount to less than one part in 700 of the complete term in $\delta\rho^2$, if the coefficients are taken from the data of Amagat.[6]

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6. E. H. Amagat, Annales de Chimie et Physique 29, 505 (1893)

Consequently, to second order eqn. (7) becomes

$$-\frac{4}{3}\mu\frac{du}{dx} = \frac{\rho_1}{a}\left[1 + \frac{\rho_1}{2c_0^2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S\right]\left[c_0^2\delta\rho^2 - \Delta p\,\delta\rho\right], \tag{10}$$

so that

$$L = \frac{16\mu c_0}{3\Delta p\left[1 + \frac{\rho_1}{2c_0^2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S\right]} \tag{11}$$

The coefficient $(\partial^2 p/\partial\rho^2)_S$ has not been determined for any of the substances studied. It could best be obtained from a study of the variation of the velocity of sound with pressure and temperature, or with density and temperature, since it is given by either of the following two relations:

$$\frac{\rho_1}{2c_0^2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S = \rho_1 c_0\left(\frac{\partial c_0}{\partial p}\right)_T + \frac{1}{\rho_1 c_0 C_v}\left(p - \frac{T\alpha}{\beta}\right)\left(\frac{\partial c_0}{\partial T}\right)_p$$

$$= \frac{\rho_1}{c_0}\left(\frac{\partial c_0}{\partial \rho}\right)_T + \frac{1}{\rho_1 c_0 C_v}\left(p - \frac{T\alpha}{\beta}\right)\left(\frac{\partial c_0}{\partial T}\right)_\rho ,$$

where in addition to the symbols previously defined, α is the coefficient of thermal expansion and β is the compressibility. Sufficient data to use these relations were not available in the literature.

However, as a rough approximation to $(\partial^2 p/\partial \rho^2)_S$ one may use Cp/Cv $(\partial^2 p/\partial \rho^2)_T$, which is available from the compressibility measurements of Amagat. In doing this it should be noted that Cp/Cv $(\partial p/\partial \rho)_T$ taken from this source differs from C_o^2 by 11% for ethyl alcohol, 20% for acetone and 36% for ethyl ether. There may also be corresponding errors in the second derivative. The expected shock front thickness is estimated for a number of substances in Table I.

TABLE I

Theoretical Shock Thickness in Acetone, Ethyl Alcohol and Ethyl Ether

Material	C_o[7] Cm.sec.$^{-1}$	μ [8] gm.cm.$^{-1}$sec.$^{-1}$	C_p/C_v[7]	$\left(\frac{\partial^2 p}{\partial \rho^2}\right)_T$ [6] dyne cm.^{4}gm.$^{-2}$	ρ_1 gm.cm.$^{-3}$	$L\Delta p$ atmospheres x cm.
Acetone	1.27×10^5	$.30 \times 10^{-2}$	1.43	4.8×10^{10}	.791	7.6×10^{-4}
Ethyl Alcohol	1.17×10^5	1.72×10^{-2}	1.13	14×10^{10}	.789	18×10^{-4}
Ethyl Ether	1.00×10^5	$.233 \times 10^{-2}$	1.33	7.9×10^{10}	.713	2.6×10^{-4}

7. E. B. Freyer, J. C. Hubbard and D. H. Andrews, J. Am. Chem. Soc. 51, 759 (1929).

8. Thorpe and Roger, Philos. Trans. A., 185, 397 (1895).

In interpreting Table I it must be realized that in using the ordinary shear viscosities no account has been taken of the possibility that μ depends on the rate of shear. Because compression in the shock is equivalent to sonic processes of the order of 300 megacycles/sec., this is not necessarily a safe assumption. All of the values listed in Table I were taken at 20°C, except the compressibility data for acetone. This was available only at 0°C.

The Effect of Bulk Viscosity on the Thickness

In recent work in the field of ultrasonic absorption, the validity of using only the shear viscosity μ to describe the dissipative forces in a dynamic system has been questioned.[9,10,11,12]

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9. L. Tisza, Phys. Rev. 61, 531 (1948)

10. Carl Eckart, Phys. Rev. 73, 68 (1948)

11. J. J. Markham, R. T. Beyer and R. B. Lindsay, Rev. Mod. Phys. 23, 353 (1951)

12. L. Hall, Phys. Rev. 73, 775 (1948)

Tisza has proposed that in addition to μ, a bulk viscosity may be effective under dynamic conditions. If k is the bulk viscosity, Eq. (11) becomes

$$L' = \left(\frac{4}{3}\mu + k\right) \frac{4c_0}{\Delta p\left[1 + \frac{\rho_1}{2c_0^2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S\right]} \qquad (12)$$

If the value of k/μ is of the order of 3, as has been reported[11] for some liquids, L' would be about 3L. Accurate experimental measurements of L combined with a well established value of $\left(\partial^2 p/\partial \rho^2\right)_S$ should yield information as to the bulk viscosity forces acting in the shock process.

Experimental Method

In order to apply the reflectivity method, a knowledge of the variation of index of refraction, n, with distance through the shock front is necessary. This may be obtained by integrating Eq. (10) to obtain

$$\rho(x) = \rho_1 + \frac{\Delta\rho}{1 + e^{-4x/L}} \quad , \tag{13}$$

and since $\delta n(x)$ is a linear function of $\delta\rho(x)$

$$n(x) = n_1 + \frac{\Delta n}{1 + e^{-4x/L}} \quad . \tag{14}$$

It is interesting to note that this is exactly the expression used by Cowan and Hornig as their "Model II" function. The reflectivity equation is therefore the same as that of Cowan and Hornig:

$$R = \frac{\Delta n^2}{4 n_1^2}(1 + \tan^4\theta)\left[\frac{\left(\frac{\pi^2 L \cos\theta}{\lambda}\right)}{\sinh^2\left(\frac{\pi^2 L \cos\theta}{\lambda}\right)}\right] \tag{15}$$

In the above equation R is the reflectivity, λ the wave-length of the incident light, θ the angle of incidence and n_1 the index of refraction of the medium. Δn is the total change in index of refraction across the shock front. The part of the equation enclosed in square brackets is plotted as a function of $\lambda/L\cos\theta$ in Fig. 1.

Two essentially different methods of determining L from Eq. (15) suggest themselves. First, following Cowan and Hornig, the parameter $L\cos\theta/\lambda$ may be varied by changing either λ or θ , and the slope of the reflectivity function determined. The value of L may then be determined from Fig. 1. Alternately, the apparatus may be calibrated in terms of absolute reflectivity and Eq. (15) solved for L. In this case n is given by the Lorentz-Lorenz formula which is, for sufficiently weak shocks,

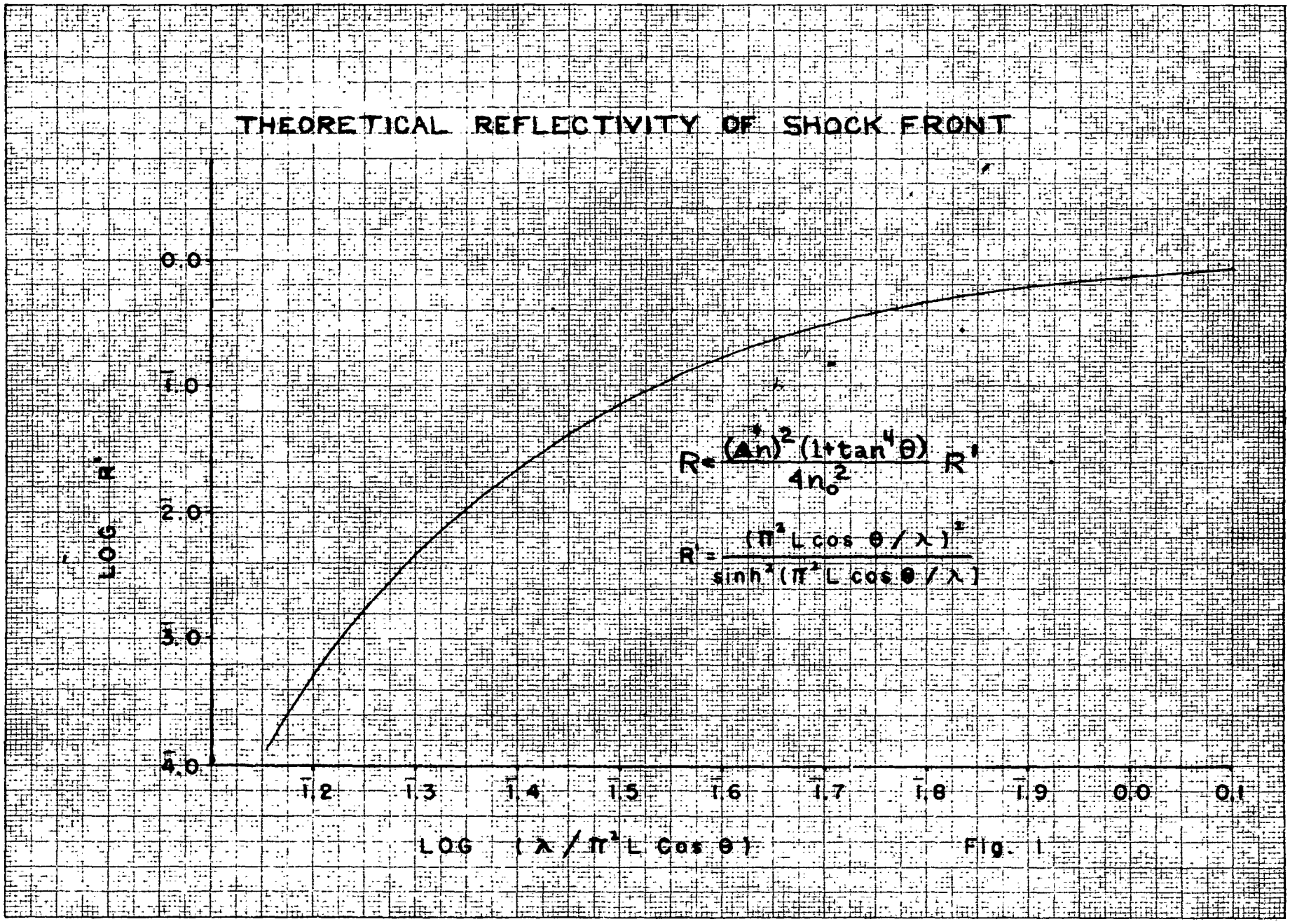

Fig. 1

$$\Delta n = \frac{\Delta \rho}{2 n_1 \rho_1} (n_1^2 - 1) \qquad (16)$$

Apparatus

A photograph of the apparatus used is shown in Fig. 2, and a schematic diagram in Fig. 3. The apparatus was a modification of that used by Cowan and Hornig. A shock wave was generated in nitrogen by means of a bursting diaphragm in a vertically mounted, 2-7/8" diameter shock tube. Copper diaphragms, .008" thick, were used to produce shocks with a pressure difference of 10 atmospheres. The shock wave travelled down the tube until it impinged at normal incidence on the surface of the liquid in the bottom of the tube. A .001" membrane of duPont "Mylar" film was located about a centimeter below the surface of the liquid and served to separate contaminated liquid above from the clean, carefully filtered sample in a cell at the base of the tube. The 3 cm. layer of liquid above the cell tended to slow down copper fragments and protect the membrane. Windows in opposite sides of the cell allowed the collimated light beam from a carbon arc to enter the liquid at a known angle to the vertical; a type 931A photomultiplier in conjunction with an optical system received the reflected light pulse as the shock travelled past the window. Suspended matter in the sample had to be carefully eliminated since it caused intolerable light noise; in fact, the reduction of scattered light was one of the major experimental problems in this work. Samples were filtered through three fine sintered glass filters directly into the cell, which was constructed entirely of stainless steel, to eliminate suspended particles as far as possible.

The shock tube was adjusted to within 2 minutes of arc of the vertical by means of a plumb bob, and the optical zero was fixed within 30 seconds of horizontal using surveying instruments. Accurate angle settings were imperative since the angles of incidence used were close to grazing, where the $(1 + \tan^4\theta)$ factor in Eq.(10) changes rapidly with angle. Shock pressures were computed from the measured pressures in the compression and expansion chambers, using previously published relations.[13]

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13. F. W. Geiger and C. W. Mautz, "The Shock Tube as an Instrument for the Investigation of Transonic and Supersonic Flow Patterns", U. S. Navy Department of Naval Research Contract No. N6-ONR-232 Task Order IV, Engineering Research Institute, University of Michigan, June, 1949, p.12.

A correction was made for the 7% decrease in shock strength, due to non-ideal bursting conditions, reported by Bleakney et al,[14]

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14. W. Bleakney, D. K. Weimer and C. H. Fletcher, Rev. Sci. Instr. 20, 807 (1949).

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as well as for the more than two-fold increase[15]

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15. R. Courant and K. O. Friedrichs, "Supersonic Flow and Shock Waves", (Interscience Publishers, Inc., New York, 1948) p. 153.

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in shock pressure caused by reflection at the liquid-gas interface.

An image of the carbon arc light source was cast on a horizontal slit, which then became the effective light source for the rest of the system. The slit was designed so that the divergence of the

beam in the vertical plane was limited to plus or minus 30 minutes of arc. The optical system was arranged so that the direct light beam passed through the cell and left through the exit window; it was absorbed on a black paper mask outside the tube. This arrangement was found to materially reduce the background of scattered light. A lens and mask in the exit system permitted only nearly parallel light to reach the photomultiplier, still further reducing the unwanted scattered rays. The entire optical system was mounted on a 500 lb. concrete block, entirely separate from the shock tube, to prevent vibrations from reaching the detection system.

The scattering level in the cell was measured before each shot by setting the angle of incidence at a standard value, and measuring the signal obtained when the arc beam was "chopped" by a rotating sector. Whenever the scattering level increased following a shot a pin hole was found in the membrane. High scattering levels gave rise not only to random "light" noise but also to well defined spurious signals.

Effective angles of incidence in the liquid were computed by using the known divergence of the light beam in connection with the reflectivity equation to obtain a properly weighted average, after allowance had been made for refraction at the windows. The external angles of incidence used were 80° and 82°, corresponding to actual effective values, in acetone and ethyl alcohol, of 82°45' and 84°15'. In ethyl ether the effective values were 82°37' and 84°05'.

Wavelengths were isolated at two points in the visible spectrum by Wratten filters; effective wavelength values were fixed by combining filter transmissions, measured using a carbon arc source in connection with a recording spectrograph, with the theoretical

reflectivity curve to obtain a weighted average value. The detector in the spectrograph was a type 1P21 photomultiplier, which has the same spectral sensitivity as the 931-A. Wratten filter No. 35 yielded a weighted average wavelength of 4300Å, based on a shock thickness of 10^{-4}cm. Wratten No. 21 yielded an average value of 5700Å. These two filters, as well as the white light average of 4750Å, were used in all the experimental work.

Output signals were displayed on a cathode ray oscilloscope and recorded photographically. An expanded sweep allowed accurate timing measurements to be made. The frequency response of the electronic system was constant to 100 kc., falling off to 50% at 400 kc.

A signal was observed in some liquids upon passage of the primary shock wave. A second signal was observed as the shock was reflected from the bottom plate of the cell and again traversed the light beam. These signals were identified as true reflections by their timing, their shape, their duration and especially by their wavelength dependence. The signals were observed to be stronger at longer wavelengths, eliminating any possibility that they might be caused by any spurious scattering phenomenon.

The recording system was calibrated in terms of absolute reflectivity by using a shock front of known thickness in nitrogen as a reference. A shock wave which had been thoroughly studied by Greene et al[3] was generated in the tube by duplicating their conditions. The thickness of this shock was known to be 1.63×10^{-5} cm. plus or minus 15%; its reflectivity was established through the use of the Rankine-Hugeniot relation in conjunction with Eq. (15) and (16). $\lambda/\pi^2 L\cos\theta$ for this shock falls to the right on Fig. 1,

where the reflectivity varies slowly with thickness, thus enhancing the accuracy of the calibration. The thicknesses observed in liquids, on the other hand, correspond to points on the left of Fig. 1, in a region of rapidly varying reflectivity, which tends to increase the reliability of the results.

The scatter of reflectivity values obtained from gas calibration shocks was considerably less than that observed in the liquid; this was originally attributed to the complications introduced by the passage of the shock through the liquid-gas interface as well as the factors mentioned in the preceeding paragraph. However, the condition of this interface at the instant of arrival of the shock wave was investigated by totally reflecting a light beam from the surface and observing any deflections. No disturbances were observed in the central portion of the surface prior to the arrival of the shock, although there was some evidence of deformations near the edges.

Experimental Results

The experimental results at $26\pm3°C$ are listed in Table II. Shock wave thicknesses were successfully measured in acetone, ethyl alcohol and ethyl ether, but no reflections could be observed in water, carbon tetrachloride, benzene or ethylene glycol, even from the strongest shock waves ($\Delta p = 16.7$ atmospheres) available. The minimum reflectivity observable with this apparatus was about $R=10^{-7}$; this fact establishes a lower limit on the shock front thickness in these liquids, as indicated in the table. The thickness values listed in Table II represent the average of 4 or 5 reflectivity measurements at each of the angle or wavelength settings, except as noted. Probable limits were computed from the scatter in experi-

TABLE II

Experimental Shock Thicknesses at 26°C.

Liquid	Shock Strength (atmospheres)	Method	Thickness ($cm. \times 10^4$)	Ave. Thickness ($cm. \times 10^4$)
Acetone	9.3	Angle dep.	1.40 ± 0.2	1.40
		Angle dep.	1.35 ± 0.2	
		Absolute mag.	1.16 ± 0.08	
		Absolute mag.	1.25 ± 0.07	
		Wave length dep.	1.54 ± 0.2	
	13.1	Angle dep.	$.74 \pm 0.15$	0.80
		Angle dep.	$.70 \pm 0.4$	
		Absolute mag.	$.90 \pm 0.07$	
		Absolute mag.	.82*	
		Wave length dep.	$.85 \pm 0.12$	
Ethyl Alcohol	13.1	Wave length dep.	1.00 ± 0.15**	1.16
		Absolute mag.	1.32**	
	16.7	Angle dep.	$.90 \pm 0.2$	1.11
		Absolute mag.	1.24 ± 0.10	
		Absolute mag.	1.18 ± 0.08	
Ethyl Ether	9.3	Wave length dep.	$.90 \pm 0.15$	0.85
		Angle dep.	1.04 ± 0.08	
		Absolute mag.	$.67 \pm 0.07$	
		Absolute mag.	$.80 \pm 0.08$	
Water	16.7	Absolute mag.	>1.1	
Carbon Tetrachloride	16.7	Absolute mag.	>2	
Benzene	16.7	Absolute mag.	>1.9	
Ethylene Glycol	16.7	Absolute mag.	>1.6	

* Two records
** One record

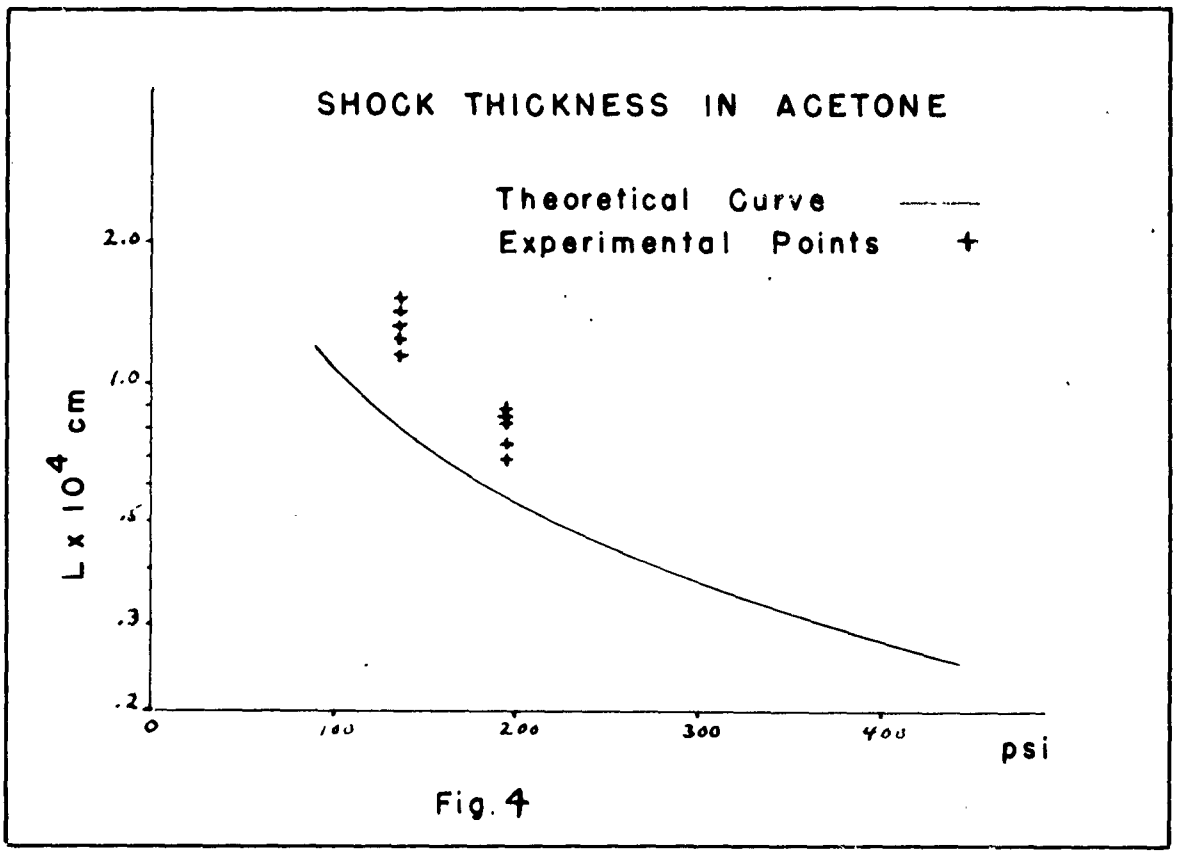
mental values. This scatter was relatively large, possibly reflecting uncertain conditions at the liquid-gas interface at the instant of arrival of the shock. The probable errors of reflectivity values ranged from 11% to 35% with and average value of 15%. The scatter in gas calibration reflectivities yielded a probable error of only 1.6%.

TABLE II

It will be noted that while there appears to be some discrepancy between the results of the various methods of measuring L, in most cases the limits of error overlap for identical shocks. The experimental results are plotted in Figures 4, 5 and 6 together with the corresponding theoretical curves computed from Eq. (9).

Conclusions

The comparison between the experimental and theoretical results for acetone and ethyl ether suggests that either the shear viscosity is markedly greater under these conditions or that a bulk viscosity effect is present, since the measured values of L all lie significantly above the predicted values. The values of $(\partial^2 p/\partial \rho^2)_S$ used in the theoretical computations are so questionable, however, that no clear cut conclusions may be drawn regarding the magnitude of the effect. The potentiality of the method for gaining information as to the viscous behavior of liquids has, however, been demonstrated; an expression for shock thickness in liquids, valid for weak shocks, has been derived from hydrodynamic theory, and an apparatus to measure the thickness of such shock waves has been described. The authors wish to thank Dr. Edward F. Greene for valuable help and suggestions throughout the experimental work and Mr. Thomas L. Dawson for help in designing and constructing the apparatus.


SHOCK THICKNESS IN ACETONE
Theoretical Curve
Experimental Points +
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.3
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0
100
200
300
400
psi

Fig. 4

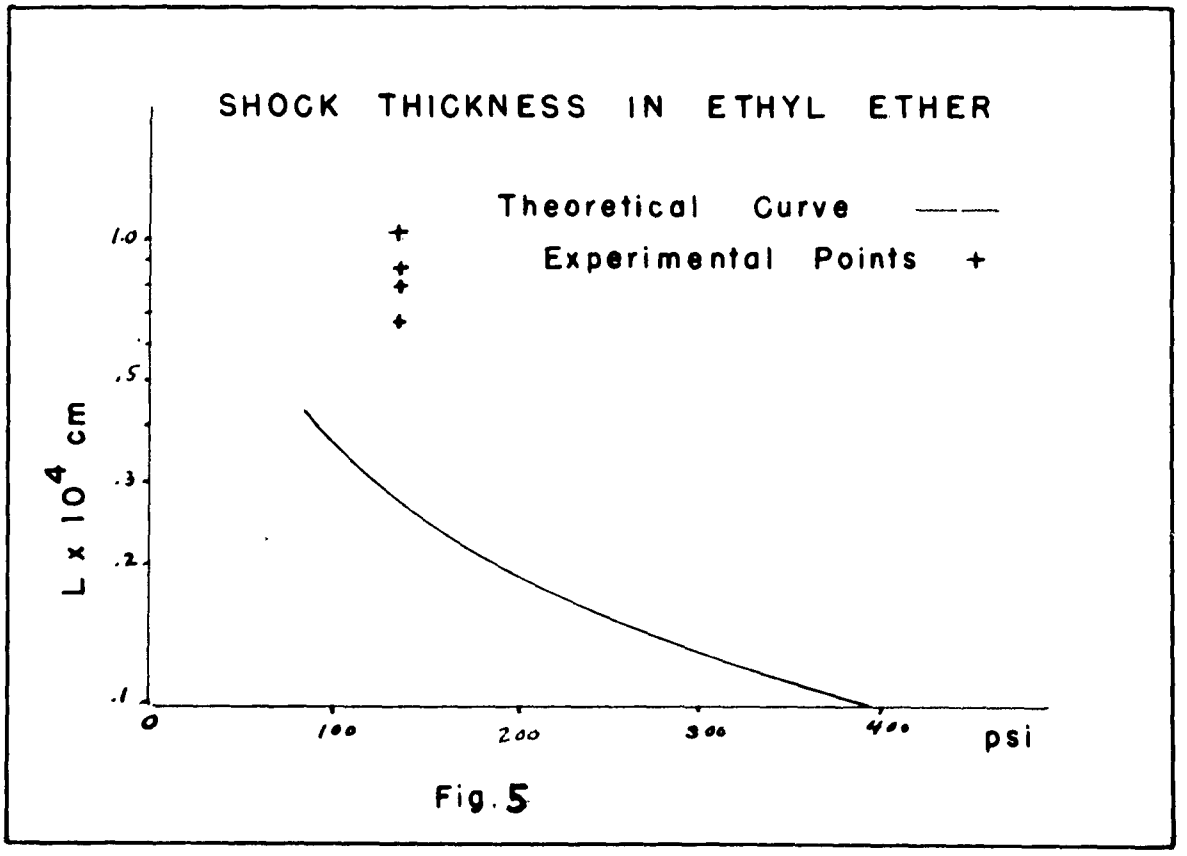
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Fig. 5

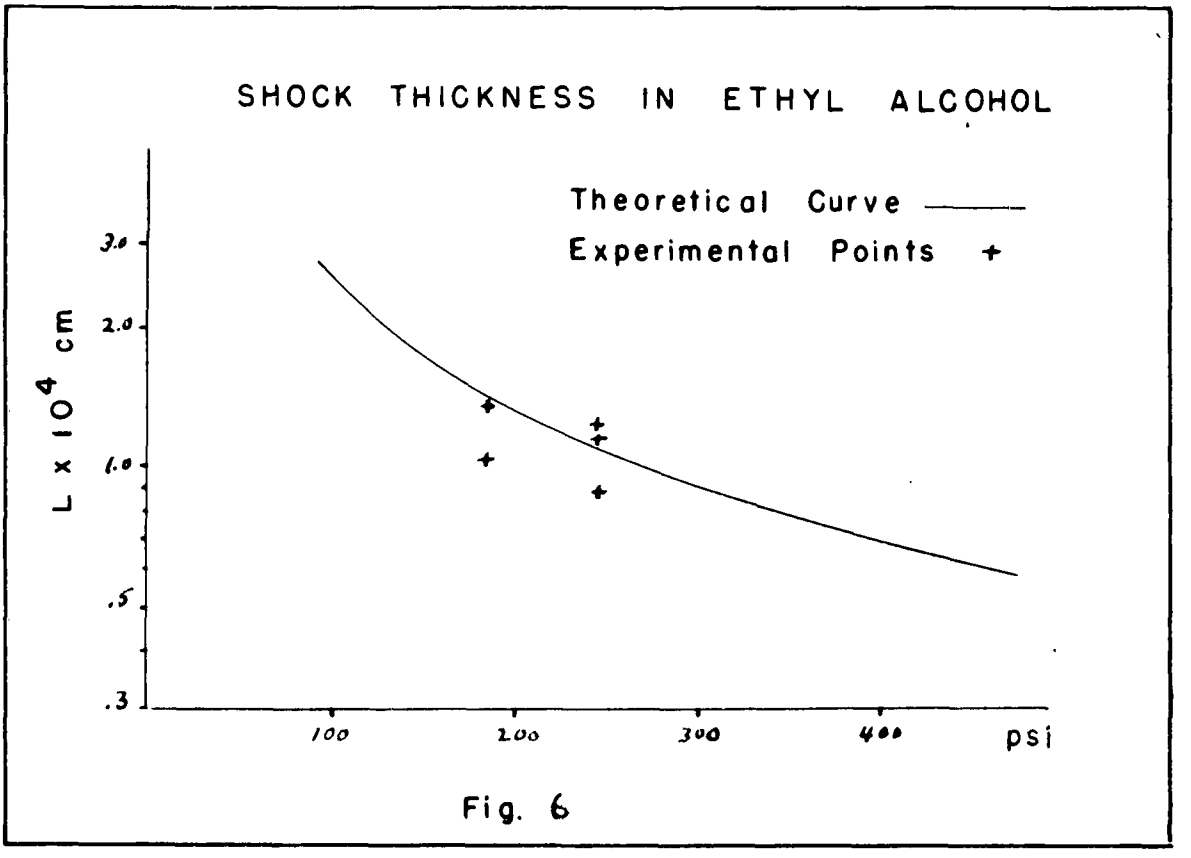
SHOCK THICKNESS IN ETHYL ALCOHOL
Theoretical Curve
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Fig. 6